

# C.U.SHAH UNIVERSITY

## Summer Examination-2016

**Subject Name : Engineering Mathematics-I**

**Subject Code : 4TE01EMT1**

**Branch : B.Tech(All)**

**Semester : 1**

**Date :21/04/2016**

**Time :10:30 To 1:30**

**Marks : 70**

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1**

**Attempt the following questions:**

**(14)**

- a) If  $z = re^{i\theta}$ , then  $|re^{iz}| =$
- a)  $e^{r\sin\theta}$       b)  $e^{-r\sin\theta}$       c)  $e^{-r\cos\theta}$       d)  $e^{r\cos\theta}$
- b) The Imaginary part of Complex number  $e^{3z}$  is
- a)  $e^y \sin x$       b)  $e^x \cos y$       c)  $e^{3x} \cos 3y$       d)  $e^{3x} \sin 3y$
- c)  $\lim_{x \rightarrow 0} \frac{\cos x}{x} = \underline{\hspace{2cm}}$ .
- a) 0      b) 1      c)  $\infty$       d)-1
- d)  $\lim_{x \rightarrow 0} \frac{x - \sin x}{x} = \underline{\hspace{2cm}}$ .
- a) 0      b) 1      c)  $\infty$       d)-1
- e) The series  $\sum \frac{1}{n}$  is
- a) Convergent      b) Divergent      c) non-convergent      d) a & b both
- f) The series  $\sum_{n=1}^{\infty} \frac{3n-5}{11n^2+2}$  is
- a) Convergent      b) Divergent      c) non-convergent      d) None of these
- g) If the power of x & y both are even ,then the curve is symmetrical about
- a) X-axis      b) Y-axis      c) about both X & Y axes      d) None of these
- h) If the two tangents at the point are real & distinct, the double point is called
- a) a node      b) a cusp      c) a conjugate point      d) None of these
- i) The series  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$  represents expansion of
- a)  $\sin x$       b)  $\cos x$       c)  $\cosh x$       d)  $\sinh x$
- j) If  $y = \cos^{-1} x$ , then  $x = \dots$
- a)  $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$       b)  $1 + \frac{y^2}{2!} + \frac{y^4}{4!} + \dots$       c)  $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$       d) None of these



- k)** If  $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \dots$
- a)  $u$                       b)  $2u$                       c)  $\tan u$                       d)  $\sin u$
- l)** If  $u = y^x$  then  $\frac{\partial u}{\partial x}$  is
- a)  $xy^{y-1}$                       b)  $0$                       c)  $y^x \log x$                       d) None of these
- m)** If  $x = r \cos \theta, y = r \sin \theta, z = z$  then  $\frac{\partial(x, y, z)}{\partial(r, \theta, z)} = \dots$
- a)  $\frac{1}{r}$                       b)  $r^2 \sin \theta$                       c)  $r$                       d)  $r^2 \cos \theta$
- n)** If  $\frac{\partial(u, v)}{\partial(x, y)} * \frac{\partial(x, y)}{\partial(u, v)} = \dots$
- a)  $0$                       b)  $-1$                       c)  $1$                       d) None of these

**Attempt any four questions from Q-2 to Q-8**

**Q-2                      Attempt all questions                      (14)**

**A**                      i) Find modulus and principal argument of  $z = \frac{1-7i}{(3+4i)}$ .                      (03)

ii) Solve the equation  $z^2 - (5+i)z + 8+i = 0$ .                      (04)

**B**                      Find and plot all the roots of  $(1+i)^{\frac{1}{3}}$ .                      (07)

**Q-3                      Attempt all questions                      (14)**

**A**                      i) Evaluate:  $\lim_{x \rightarrow 0} \left[ \frac{a^x + b^x + c^x}{3} \right]^{\frac{1}{x}}$ .                      (04)

ii) Find Maclaurin's Series of  $f(x) = \cos x$ .                      (03)

**B**                      i) Show that  $f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}; & (x, y) \neq (0,0) \\ 0 & ; (x, y) = (0,0) \end{cases}$  is continuous at every point                      (04)

except at the origin.

ii) Evaluate:  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^2 \tan x}$ .                      (03)

**Q-4                      Attempt all questions                      (14)**

**A**                      Trace the curve (Cisoid of Diocle)  $y^2(2a-x) = x^3$ .                      (07)

**B**                      Find the Taylor's series expansion of  $f(x) = \tan x$  in powers of  $\left(x - \frac{\pi}{4}\right)$  showing                      (07)

at four nonzero terms. Hence, find the value of  $\tan 45^\circ$ .



- Q-5** **Attempt all questions** (14)
- A** Trace the curve (**Cardioid**)  $r = a(1 + \cos \theta)$ . (07)
- B**
- i) Test the convergence of the series  $\frac{2}{1} + \frac{3}{8} + \frac{4}{27} + \dots + \frac{n+1}{n^3} + \dots$  (04)
- ii) Test the convergence of the series (03)
- $$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} + \dots$$
- Q-6** **Attempt all questions** (14)
- A**
- (i) Test the convergence of the series  $\frac{1}{1+3} + \frac{2}{1+3^2} + \frac{3}{1+3^3} + \dots$  (05)
- (ii) Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ . (02)
- B** Find the radius of convergence & interval of convergence of the series (07)
- $$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$$
- Q-7** **Attempt all questions** (14)
- A**
- (i) If  $u = \log(x^3 + y^3 - x^2y - xy^2)$ , then Prove that  $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = -\frac{4}{(x+y)^2}$ . (05)
- (ii) Find the values of  $\frac{\partial f}{\partial x}$  &  $\frac{\partial f}{\partial y}$  at the point (4,-5).if  $f(x, y) = x^2 + 3xy + y - 1$ . (02)
- B**
- (i) If  $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x} + \sqrt{y}}\right)$ . Prove that (07)
- a)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$
- b)  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} (\tan^3 u - \tan u)$ .
- Q-8** **Attempt all questions** (14)
- A**
- (i) Find Maxima & Minima of the function  $x^3 + y^3 - 3x - 12y + 20$ . (05)
- (ii) If  $x = r \cos \theta$ ,  $y = r \sin \theta$  then find  $\frac{\partial(x, y)}{\partial(r, \theta)}$ . (02)
- B**
- (i) Expand  $f(x, y) = e^x \cos y$  in powers of x & y up to second degree. (05)
- (ii) Find the equations of tangent plane & normal line at the point (-2,2,-3) to the ellipsoid  $\frac{x^2}{4} + y^2 + \frac{z^2}{9} - 3 = 0$  (02)

